MATH 1920, Fall 2017
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Sections: 201, 202

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Section 15.3–15.5 Review

Section 15.3 Additional Exercises

Compute the first order partial derivatives.

1. $z = \ln(x^2 + y^2)$

2. $z = \tan(x/y)$

3. Use the linear approximation to estimate the percentage change in volume of a right circular cone of radius $r = 40$ cm if the height is increased from 40 to 41 cm.

4. Show that the following functions are harmonic:

$$u(x, y) = e^x \cos y$$

$$u(x, y) = \ln(x^2 + y^2)$$

5. Show that if $u(x, y)$ is harmonic, then the partial derivatives $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ are harmonic.

Section 15.4 Additional Exercises

Find an equation of the tangent plane at the given point.

1. $x^2y + xy^3$, $(1, 2)$

2. $\sin(uw)$, $(\frac{\pi}{6}, 1)$

3. Find the points on the graph of $z = 3x^2 - 4y^2$ at which the vector $n = \langle 3, 2, 2 \rangle$ is normal to the tangent plane.

Section 15.5 Additional Exercises

1. Use the chain rule to calculate $\frac{d}{dt} f(r(t))$ for $f(x, y) = 3x - 7y$, $r(t) = \langle \cos t, \sin t \rangle$, $t = 0$.

2. Calculate the directional derivative of $f(x, y, z) = xe^{-yz}$, in the direction of $v = \langle 1, 1, 1 \rangle$, at $P = (1, 2, 0)$. (Note: there is a typo in the solutions, this is a function of three variables)

3. A bug located at $(3, 9, 4)$ begins walking in a straight line toward $(5, 7, 3)$. At what rate is the bug's temperature changing if the temperature is $T(x, y, z) = xe^{y-z}$? Units are in meters and degrees Celsius. Which direction should the bug walk to experience the maximum rate of change in temperature possible? Which direction should the bug walk to experience the minimun rate of change in temperature possible?

4. Let $f(x, y) = \tan^{-1} \frac{x}{y}$ and $u = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$.
   (a) Calculate the gradient of f.
   (b) Calculate $D_u f(1, 1)$ and $D_u f(\sqrt{3}, 1)$.
   (c) Show that the lines $y = mx$ for $m \neq 0$ are level curves for f.
   (d) Verify that ∇f_P is orthogonal to the level curve through P for $P = (x, y) \neq (0, 0)$.

5. Find a vector normal to the surface $x^2 + y^2 - z^2 = 6$ at the point $P = (3, 1, 2)$.

6. Find the two points on the ellipsoid

$$\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$$

where the tangent plane is normal to $v = \langle 1, 1, -2 \rangle$.